Vortical Singularity behind a Highly Yawed Cone

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The occurrence of the vortical singularity away from the cone surface is examined using the sign of v_θ (rate of change of normal velocity in a direction normal to the body surface) at the body surface. The vortical singularity can only occur away from the body surface if the cross flow is locally supersonic. Two different categories are defined depending on whether v_θ first becomes positive when the surface cross flow is subsonic or supersonic. If v_θ becomes positive when the cross flow is supersonic, and remains positive, then an internal shock must occur. For a narrow band of conditions an internal shock will occur with v_θ negative. A numerical method, which permits discontinuous solutions at the internal shock and at the vortical singularity, is used to obtain results to support the above conclusions. The numerical method, based partly on the work of Gilinskii, Telinin, and Tinyakov (GTT) and partly on the method of characteristics, produces solutions which indicate that the vortical singularity is most likely to occur outside the leeward separated region if both the incidence and cone angle are as large as possible.

Nomenclature

a = local sound speed
a^* = critical sound speed
c_p = pressure coefficient, $c_p = (p - p_\infty) / \frac{1}{2} U_\infty^2$
M = Mach number
M_{cr} = cross flow Mach number, $M_{cr} = \{(v^2 + w^2)/a^2\}^{1/2}$
M_r = radial Mach number, $M_r = u/a$
M_∞ = freestream Mach number
p = nondimensional pressure, $p = p_{\text{dim}}/\rho_\infty a^{*2}$
p_o = nondimensional stagnation pressure
r = radial coordinate
u = nondimensional radial velocity component, $u = u_{\text{dim}}/a^*$
v = nondimensional normal velocity component
w = nondimensional circumferential velocity component
α = angle of attack
γ = specific heat ratio
θ = angular coordinate measured from the cone axis
θ_b = cone nose angle, location of body surface
θ_s = angular location of the outer shock
ρ = nondimensional density, $\rho = \rho_{\text{dim}}/\rho_\infty$
ϕ = circumferential coordinate measured from the windward line of symmetry
ϕ_{is} = internal shock location
ϕ_{sep} = circumferential location at which the flow separates from the surface of the cone

Subscripts

b = conditions at body
o = stagnation conditions
s = conditions at shock
∞ = freestream conditions
θ, ϕ = partial differentiation with respect to these quantities

I. Introduction

THE numerical study of supersonic flow about inclined cones[1-12] has generally been limited to moderate angles-of-attack $\alpha/\theta_b \lesssim 1$. For such conditions the entropy has a large gradient normal to the cone surface, and at the intersection of the leeward symmetry plane with the cone surface the entropy is multivalued.[13] This point of multivalued entropy is called the vortical singularity. The entropy (or vortical) layer at the surface has been extensively studied.[13-18] In these studies it has been postulated[13] that at sufficiently large angle of attack the vortical singularity leaves the surface. Melnik[16] obtained a criterion for lift-off of the vortical singularity. The numerical studies of Jones[3] qualitatively support Melnik's criterion. Previous numerical results[3,5] that indicate the occurrence of the vortical singularity away from the body have been marred by an inability to permit discontinuous solutions at the vortical singularity.

The vortical singularity arises as a result of the assumption that the flow is both conical and inviscid. The vortical singularity is the point (in the cross flow plane) at which all the stream surfaces meet when projected into the cross flow plane (Fig. 1). Figure 1 is based on the numerical results produced by the method to be described in Sec. III. An alternative definition of the vortical singularity location is that point on the leeward line of symmetry for which $v = w = 0$ away from the body surface (see Fig. 2 for the definition of v and w). Since entropy remains constant along all streamlines except when crossing a shock wave, the entropy is necessarily multivalued at the vortical singularity.

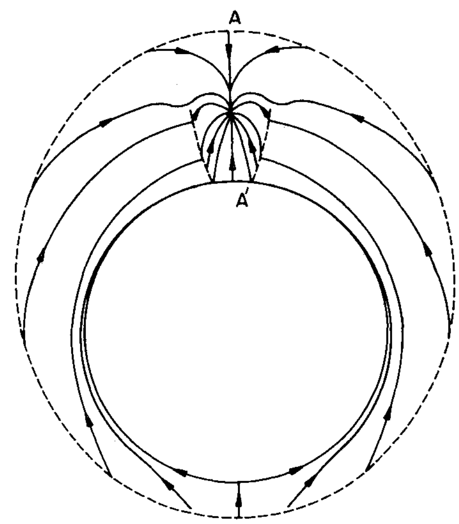

Fig. 1 Streamline pattern at $M_\infty = 7$, incidence = 30°, and nose angle = 20°.

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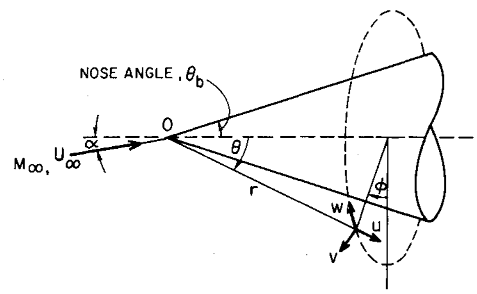

Fig. 2 Coordinate system and velocities.

This multivaluedness can best be appreciated by considering the variation of the flow variables along the leeward line of symmetry (AA' in Fig. 1). This is shown in Fig. 3. Since the leeward line of symmetry is a streamline the entropy parameter should be constant with a change (jump) in its value at the vortical singularity. As a result a discontinuity is also expected in density and radial velocity component (not shown). However, the pressure is expected to remain continuous when traversing the vortical singularity. For comparative purposes a numerical solution produced by the shock capturing method[5] is also shown in Fig. 3.

The streamline pattern shown in Fig. 1 is appropriate to inviscid flow. At angles of attack for which the vortical singularity occurs away from the body surface it is to be expected that viscous effects will predominate at least adjacent to the body surface in the leeward region. This raises the question whether there are any values of M_∞, α, and θ_b for which the vortical singularity occurs outside the leeward separated region. Indeed one of the aims of this paper is to determine just those values of M_∞, α, and θ_b, for which the vortical singularity lies furthest from the body, i.e., most likely to be outside the leeward separated region. There already exists some experimental evidence to support this idea. Feldhunh et al.[19] have measured a localized density gradient (equivalent to a density discontinuity in inviscid flow) coupled with $v = w = 0$ outside the leeward separated region, i.e., a vortical singularity. Two other experimental studies[23,26] indicate a vortical singularity-like behavior on the outer edge of the leeward separated region. However, Yahalom's[23] experiments were at $M_\infty = 2.72$, and George's[26] experiments were for $\alpha/\theta_b \approx 1$. Both these conditions would lead to a relatively small region of supersonic cross flow and the likelihood that the vortical singularity would not be far removed from the body surface.

It seems plausible that evidence of the vortical singularity occurring outside the leeward separated region will be found for values of M_∞, α, and θ_b which produce the smallest separated region. Of the many experimental studies[19-26] of inclined cones only Avduevskii and Medvedev[20] have systematically obtained the dependence of the separation angle on the 3 external parameters M_∞, α, and θ_b. For $M_\infty > 5$ and $\alpha/\theta_b > 1$ a correlation between ϕ_{sep} and θ_b is found[20] to be independent of M_∞ and α (Table 1).

Thus the smallest region of separated flow corresponds to the largest value of θ_b. The restrictions $M_\infty > 5$ and $\alpha/\theta_b > 1$ approximately conicide with the occurrence of some supersonic cross flow, which is a necessary (although not sufficient) condition for lift-off of the vortical singularity.

Since the crossflow separation is associated with the impingement of an internal shock on the boundary layer, the location of the foot of the shock adjacent to the body surface may be taken as a qualitative indication of the crossflow separation angle.[21] The present numerical study predicts that an internal shock will occur for many values of M_∞, α, and θ_b for which supersonic crossflow results. However, the location of the internal shock, predicted by any inviscid method (Fig. 4), will be poor since no account can be taken of the boundary-layer/shock interaction.

The location of the vortical singularity along the leeward

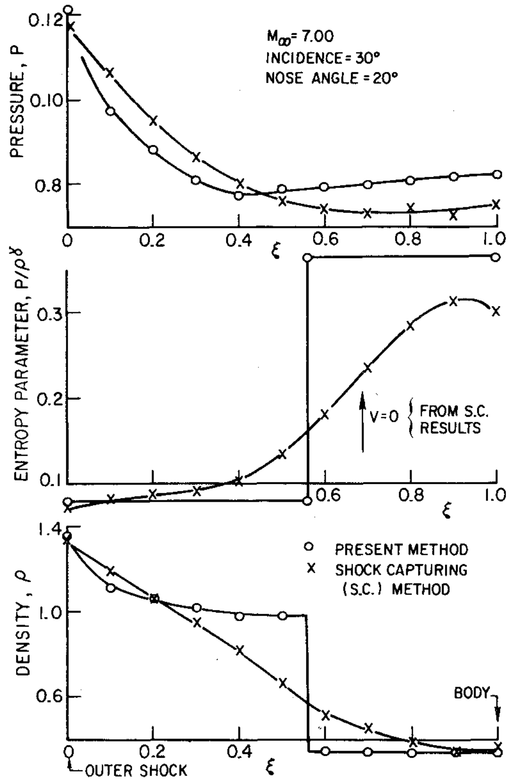

Fig. 3 Flow variation along leeward line of symmetry.

Table 1 Correlation between separation angle and nose angle

θ_b	5°	10°	15°	30°
ϕ_{sep}	118°	130°	138°	145°

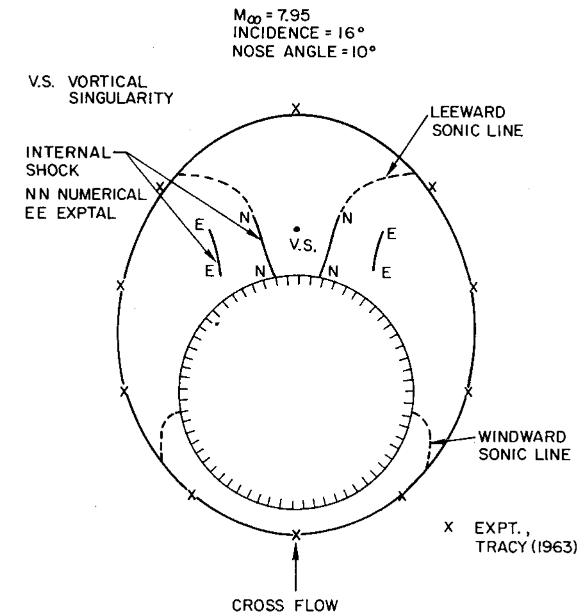

Fig. 4 Comparison with experiment—shock wave and sonic line locations.

line of symmetry may be gaged by considering the contour $v = 0$ which links the vortical singularity to the body (V.S. -A in Fig. 5). At the point where this contour intersects the body (A in Fig. 5), v_θ at the surface changes sign. Since point A lies outside the leeward separated region and in view of the hyperbolic nature of the inviscid flow it is to be expected that point A is unaffected by the leeward separated region.

In the present study, the criterion v_θ positive at the surface

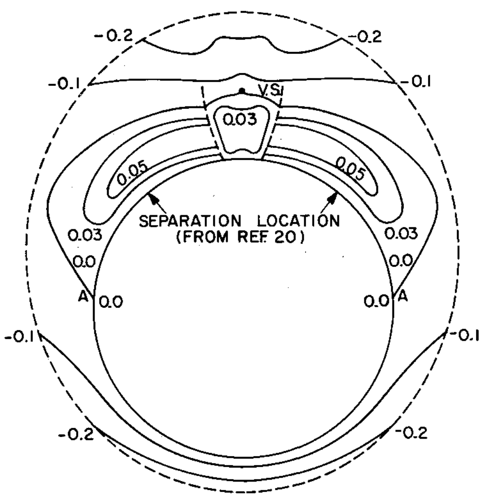

Fig. 5 Contours of normal velocity component at $M_\infty = 7$, incidence = 30°, and nose angle = 20°.

for ϕ less than 180° will be used as a criterion for the occurrence of the vortical singularity away from the cone surface (v_θ is always negative at the windward line of symmetry). Analytic expressions for the dependence of v_θ/u on $w_\phi/u \sin\theta_b$ and on M_{cr} then lead to significantly different behavior depending whether the crossflow is subsonic or supersonic when v_θ first becomes positive. A numerical method, which permits a discontinuous solution at the vortical singularity and at the internal shock, is used to provide supporting information.

II. Behavior at the Surface

In this section, the flow behavior at the surface will be considered, and it will be demonstrated that the location of the vortical singularity will be markedly changed if v_θ at the surface becomes positive when the local cross flow is supersonic rather than subsonic. For conical flow in a spherical coordinate system (r, θ, ϕ), the dependent variables u, v, w, ρ, and p are functions of θ and ϕ only. The inviscid equations of motion (continuity, three conservation of momentum, and conservation of entropy equations) have the following simple form at the surface

$$u_\phi = w \sin\theta_b \tag{1}$$

$$p_\theta = \rho w^2 \cot\theta_b \tag{2}$$

$$w w_\phi + (1/\rho) p_\phi + u w \sin\theta_b = 0 \tag{3}$$

$$v_\theta + (1/\sin\theta_b)\cdot w_\phi + (w/\rho\sin\theta_b)\cdot\rho_\phi + 2u = 0 \tag{4}$$

and

$$p_\phi = a^2 \cdot \rho_\phi \tag{5}$$

From Eqs. (3-5), ρ_ϕ and w_ϕ can be eliminated to give

$$v_\theta = -u - (p_\theta/\rho\cdot w\cdot \sin\theta_b)\;[M_{cr}{}^2 - 1] \tag{6}$$

At the surface $M_{cr} = w/a$. If the cross flow is everywhere subsonic p_ϕ only becomes positive close to $\phi = \pi$ and only then for conditions of relatively large $M_\infty \sin\alpha$ and small θ_b.[3] If the cross flow becomes supersonic p_ϕ is everywhere negative up to the termination of the supersonic cross flow region. At an internal shock p_ϕ is positive. In the leeward subsonic region (after a local region of supersonic cross flow) p_ϕ is generally positive. The quantities u, w, ρ, and $\sin\theta_b$ are always positive. Thus Eq. (6) demonstrates the change in dependence of v_θ on p_ϕ when the cross flow changes from subsonic to supersonic at the surface.

Use of Eq. (3) permits Eq. (6) to be rewritten as

$$v_\theta = -u + [u + (w_\phi/\sin\theta_b)]\,(M_{cr}^2 - 1) \tag{7}$$

The rate of change of the cross flow Mach number, M_{cr}, with ϕ is given, with the help of Eqs. (1-7), by

$$M_{cr_\phi} = \frac{\sin\theta_b}{a(M_{cr}^2 - 1)}\,[u(2 - (\frac{3-\gamma}{2})M_{cr}^2) + v_\theta(1 + \frac{\gamma - 1}{2}M_{cr}^2)] \tag{8}$$

The variation of v_θ with ϕ is given by

$$v_{\theta_\phi} = w\sin\theta_b\cdot[M_{cr}^2 - 2] - (p_\phi/\rho w\sin\theta_b)\cdot 2\cdot M_{cr}\cdot M_{cr_\phi} - [w_{\phi\phi}/\sin\theta_b]\;[1 - M_{cr}^2] \tag{9}$$

From the Bernoulli equation, one can obtain

$$(p_o/p) = [1 + (\gamma-1)/2]M_{cr}^2\;\;[(\gamma-1)/2]M_r^2]^{\gamma/(\gamma-1)} \tag{10}$$

where

$$M_{cr} = (v^2 + w^2)^{1/2}/a$$

and

$$M_r = u/a$$

Examination of Eq. (10) indicates that the cross flow can change smoothly from supersonic to subsonic without the need for the pressure to increase. The radial Mach number M_r is quite close to the freestream value M_∞, and small changes in p occur due to small changes in u and hence M_r. This behavior is relatively independent of M_{cr} which controls the character of the equations of motion.

If the flow becomes subsonic in the leeward region without an internal shock then Eq. (6) indicates that v_θ must be negative when $M_{cr} = 1$. To preserve the boundary condition of zero normal flow into the body, the internal shock must be locally normal to the body. This implies that v and v_θ are continuous through the shock at the body surface. If v_θ is positive in the supersonic cross flow region then Eq. (6) indicates that an internal shock must occur. If v_θ is negative in the supersonic cross flow region an internal shock may or may not occur. Generally an internal shock will not occur. For v_θ not to become positive in supersonic cross flow region, Eq. (6) implies that M_{cr} is small, and as indicated by Eq. (10) there is then generally sufficient 'stretch' to permit the cross flow to become subsonic without the need for an internal shock.

At $\phi = \pi$, Eq. (6) indicates

$$v_\theta = -u + (p_\phi/\rho\cdot w\cdot\sin\theta_b) \tag{11}$$

However at $\phi = \pi$, p_ϕ and $w = 0$; therefore by L'Hospital's rule

$$v_\theta = -u + \frac{p_{\phi\phi}}{\rho w_\phi \sin\theta_b} \tag{12}$$

From Eq. (4) applied at $\phi = \pi$ it follows that

$$w_\phi = -(2u + v_\theta)\,\sin\theta_b \tag{13}$$

and substitution into Eq. (12) leads to

$$v_\theta^2 + 3u \cdot v_\theta + [2u^2 + (p_{\phi\phi}/\rho \sin^2\theta_b)] = 0 \tag{14}$$

or

$$v_\theta = \left(\frac{u^2}{4} - \frac{p_{\phi\phi}}{\rho \sin^2\theta_b} \right)^{1/2} - \frac{3u}{2} \tag{15}$$

The condition that v_θ should be positive at $\phi = \pi$ leads to

$$p_{\phi\phi} < -2 \cdot \gamma \cdot p \cdot M_r^2 \cdot \sin^2\theta_b \tag{16}$$

This is precisely the condition that Melnik[16] obtained. In the present study, the equivalent expression for v_θ becoming positive at any point on the cone surface is obtained from Eq. (6) as

$$|p_\phi| > |\rho u w \sin\theta_b / (M_{cr}^2 - 1)| \tag{17}$$

From Eq. (7) the condition that $v_\theta = 0$ is given by

$$\frac{w_\phi}{u \sin\theta_b} = \frac{(2 - M_{cr}^2)}{(M_{cr}^2 - 1)} \tag{18}$$

From Eqs. (7) and (8) the condition that $M_{cr_\phi} = 0$ is given by

$$\frac{w_\phi}{u \sin\theta_b} = \frac{-1}{\left[1 + \dfrac{2}{(\gamma - 1) M_{cr}^2} \right]} \tag{19}$$

Equations (18) and (19) are plotted in Fig. 6 and used to define regions in which v_θ and M_{cr_ϕ} remain positive, etc. One may note that the criterion Eq. (16) (first given by Melnik), is equivalent to v_θ becoming positive in the lower left hand region of Fig. 6. Since v_θ first becomes positive in a region of subsonic cross flow, this will be referred to as a 'subsonic' lift-off of the vortical singularity. In contrast, the upper right hand region indicates that v_θ can readily become positive while the surface cross flow is supersonic. As long as v_θ remains positive as the leeward line of symmetry is approached it can be inferred that the vortical singularity occurs away from the body surface. This will be referred to as a 'supersonic' lift-off. The condition that v_θ remains positive with increasing ϕ is generally valid except for a very narrow band of conditions. This will be discussed in Sec. IV.

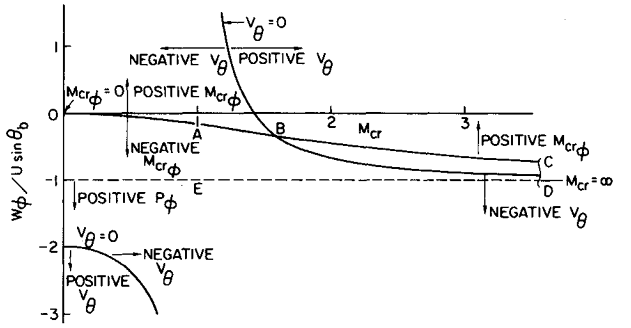

Fig. 6 Variation of the circumferential velocity gradient parameter at the surface with cross flow Mach number.

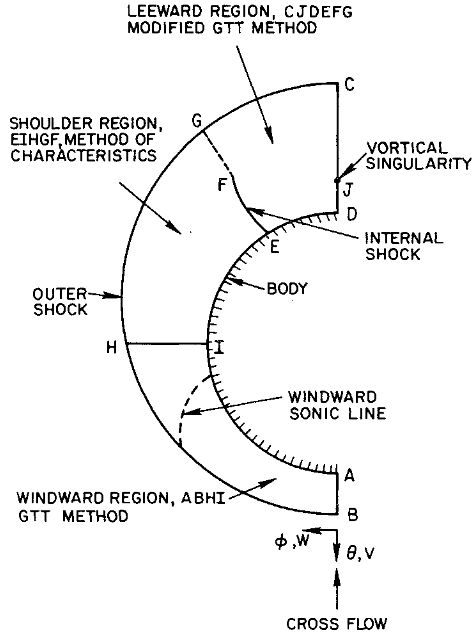

Fig. 7 Classification of the different flow regions on the basis of the numerical method used.

III. Numerical Method

To test the criterion developed in Sec. II, experimental or numerical evidence is required. A numerical method, which permits the occurrence of an internal shock and the occurence of a vortical singularity away from the surface, has been used to obtain suitable evidence; this method will now be briefly described.

The method is based on splitting the flow region into three physically distinct regions (Fig. 7). On the windward side of the cone the equations of motion are elliptic. In this region the method of Gilinskii, Telinin, and Tinyakov[27] (GTT) is followed. The equations of motion are rearranged to look like ordinary differential equations in the normal coordinate. These equations are integrated simultaneously along 5 rays from the outer shock to the body surface. An iterative technique based on a function minimization method due to Powell[28] is employed to adjust the outer shock location until the body boundary condition of zero normal flow is satisfied. The final ray of the windward region (HI in Fig. 7) is deliberately chosen to overlap the windward sonic line. Thus the solution on HI provides initial data for the method of characteristics used in the shoulder region.

In the shoulder region the upstream interpolation method of characteristics[29] has been followed; this method retains a fixed grid. Values of u, v, w, ρ, and p at a downstream mesh point are obtained by projecting the characteristic lines through the mesh point in the upstream direction until they intersect the previous line of data. At the intersections the local function values can be obtained by interpolating among adjacent points in the normal direction. The compatibility conditions then connect the upstream interpolated values of the functions with the unknown values at the downstream mesh points.

Because of the hyperbolic nature of the governing equations the supersonic crossflow has no warning of the leeward symmetry requirement. Consequently an internal shock occurs; the region behind the internal shock has a subsonic crossflow. In this region (Fig. 7) the GTT method is applied in the other direction; integration is made between the internal shock and the leeward line of symmetry.

Further description of the numerical method and a comparison with experimental results[21] and other numerical methods such as the shock-capturing method[5,30] are available elsewhere.[31,32] As an illustration, the present method agrees with the shock capturing method to within ½% for all locations outside the leeward region. The lack of agreement within the leeward region essentially follows from the results shown in Fig. 3. Because the present method takes full advantage of the conical nature of the problem, it is approximately ten times more economical of computer time than the shock-capturing method.

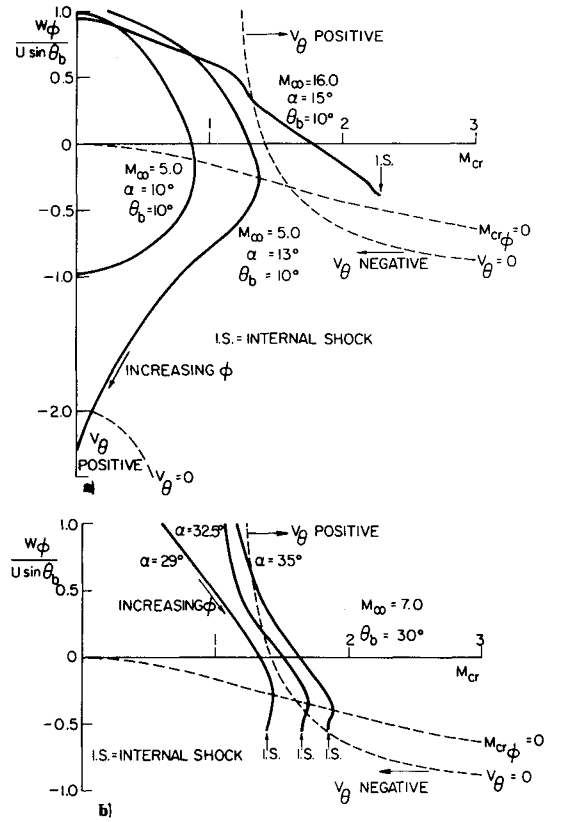

Fig. 8 Circumferential velocity gradient parameter variation for a) three characteristic cases and b) large cone angle.

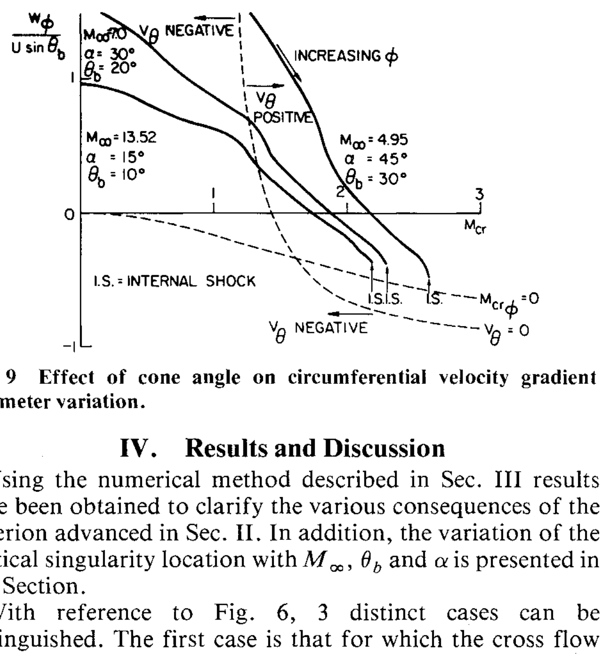

Fig. 9 Effect of cone angle on circumferential velocity gradient parameter variation.

IV. Results and Discussion

Using the numerical method described in Sec. III results have been obtained to clarify the various consequences of the criterion advanced in Sec. II. In addition, the variation of the vortical singularity location with M_∞, θ_b and α is presented in this Section.

With reference to Fig. 6, 3 distinct cases can be distinguished. The first case is that for which the cross flow remains completely subsonic (typically $\alpha/\theta_b \leq 1$ for small to moderate θ_b). A typical set of results[33] ($M_\infty = 5$, $\alpha = 10°$, $\theta_b = 10°$) is shown in Fig. 8a. Following the curve in the direction of increasing ϕ indicates that at no point does v_θ become positive, thus the vortical singularity remains attached to the surface.

The next case is that for which a small region of supersonic cross flow develops adjacent to the cone surface. This situation is only obtained for small cone angles ($\theta_b \leq 10°$). For large cone angles the cross flow first becomes supersonic at the outer shock wave. For small cone angles, the cross flow first becomes supersonic at $\alpha/\theta_b \approx 1.1$. For large cone angles, supersonic cross flow appears at smaller values of α/θ_b(e.g., $\alpha/\theta_b \approx 0.75$ at $\theta_b = 30°$.) For $M_\infty = 5$, $\alpha = 13°$, $\theta_b = 10°$,[33] the

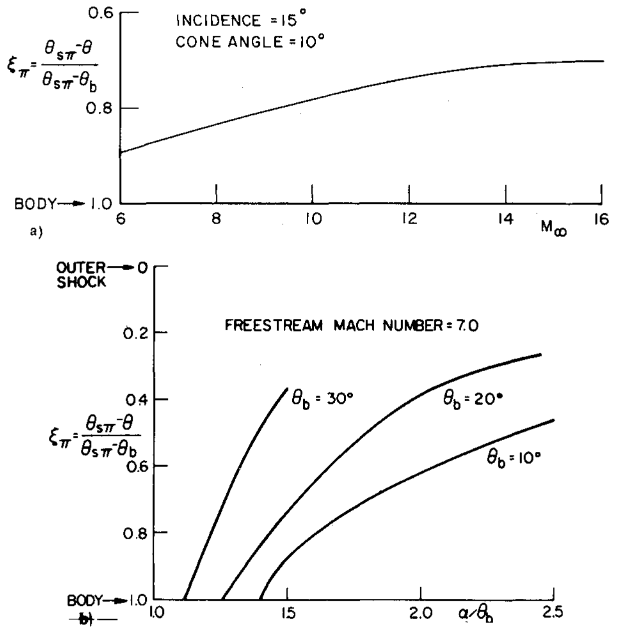

Fig. 10 Dependence of vortical singularity location on a) freestream Mach number and b) incidence and cone angle.

results are shown in Fig. 8a. After the flow becomes supersonic, it moves close to the boundary $v_\theta = 0$. However the cross flow Mach number reaches a maximum and, after the cross flow again becomes subsonic, v_θ becomes positive close to the leeward line of symmetry (lower left-hand region of Fig. 8a). The limited region of supersonic cross flow causes the flow to expand to a fairly large ϕ (typically 140°). Thus the subsequent deceleration of w leads to a large negative w_ϕ. This in conjunction with the small θ_b is sufficient to cause v_θ to become positive [Eq. (11)] close to $\phi = \pi$. This is essentially the case studied by Melnik[16] and Jones.[3]

At higher Mach numbers and angles of attack, v_θ becomes positive after the cross flow becomes supersonic. A specific example, $M_\infty = 16$, $\alpha = 15°$, and $\theta_b = 10°$, is shown in Fig. 8a. It can be seen that M_{cr} continues to grow and v_θ remains positive until an internal shock occurs. v_θ is continuous at the surface through the internal shock and remains positive until the leeward line of symmetry is reached. With reference to Fig. 8a this constitutes a continuation off the lower left-hand edge of the figure. For this case the vortical singularity occurs roughly a quarter of the way between the cone surface and the outer shock. Positive values of v_θ have been obtained for $\alpha/\theta_b > 1.5$ at small θ_b and M_∞, and for $\alpha/\theta_b > 1.2$ at large θ_b or M_∞. This typically corresponds to conditions for which an internal shock is obtained.

The change in pattern between the completely subsonic case and the locally supersonic occurs quite smoothly. However the change between the locally supersonic case and the occurrence of an internal shock and/or positive v_θ and vortical singularity lift-off is more varied. As already noted, the lift-off of the vortical singularity after a local region of supersonic cross flow only occurs for small cone angle. For large cone angle the first change in pattern, after the flow becomes supersonic at the surface, is the appearance of an internal shock. As can be seen by the example ($M_\infty = 7$, $\theta_b = 30°$, $\alpha = 29°$, Fig. 8b), v_θ remains negative throughout. With a small increase in incidence (to $\alpha = 32.5°$), v_θ becomes positive and then negative again before the internal shock occurs. For both these cases the vortical singularity remains attached to the surface. The internal shock provides a sufficiently large reduction in w that w_ϕ after the internal shock can be smaller, and hence v_θ is more likely to remain negative. With a further small increase in incidence ($\alpha = 35°$), v_θ is positive at the internal shock and remains so until $\phi = \pi$. Thus in using a positive v_θ as a criterion for indicating the vortical singularity occurs away from the body surface, care is required in the above narrow region. For large cone angles this narrow band of positive v_θ occurs for all freestream Mach numbers. For small cone angles the band of positive v_θ only occurs at large Mach numbers.

For small angles of attack and small freestream Mach number, it is found that the 3 external parameters M_∞, α, and θ_b can be conveniently grouped as $M_\infty \sin \alpha$ and α/θ_b. Thus flows at the same values of $M_\infty \sin \alpha$ and α/θ_b may be expected to be similar. This does not seem to be the case for large angles of attack and for large freestream Mach numbers, i.e., flows for which a large region of supersonic cross flow occurs. Figure 9 shows three cases that have the same $M_\infty \sin \alpha$ and α/θ_b but each case has a different cone angle, θ_b. It can be seen that increasing θ_b moves the curves to the right, i.e., at the same conditions of $M_\infty \sin \alpha$ and α/θ_b a large cone angle is more likely to cause supersonic lift-off of the vortical singularity and the occurrence of an internal shock.

The movement of the vortical singularity away from the surface with increasing Mach number is shown in Fig. 10a. However, the effect is not as great as increasing the incidence which causes the vortical singularity to lie a long way from the cone surface particularly for large cone angles (Fig. 10b). This trend is consistent with Feldhuhn's[19] measurements of the vortical singularity location.

The internal shock typically extends from one third to half way from the cone surface to the outer shock and is approximately normal to the cone surface. Thus the primary role of the internal shock is to reduce the circumferential velocity component, w. The normal orientation of the internal shock has little effect on the normal velocity component, v. The effect of increasing α, and to a lesser extent M_∞ or θ_b, causes the maximum cross flow Mach number to be higher, and hence the internal shock is stronger and closer to the leeward line of symmetry. However, the movement of the internal shock is small. For all the cases considered the internal shock has been within the range 6°-15° from the leeward line of symmetry. Because of the neglect of viscous effects agreement with the primary internal shock location obtained from experimental results is not expected.

In conclusion, a criterion for predicting the occurrence of the vortical singularity away from the body surface has been advanced. The criterion, based on the change of sign of v_θ at the body surface, indicates that the vortical singularity is likely to occur further from the body surface if the crossflow is supersonic when v_θ first becomes positive. Numerical solutions obtained to validate the criterion suggest that experimental evidence for the occurrence of the vortical singularity outside the leeward separated region is most likely to be found for θ_b and α both large.

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